Deconfinement: Renormalized Polyakov Loops to Matrix Models

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1. Review: Phase Transitions at $T \neq 0$, Lattice Results for N=3, nf = $0 \rightarrow 3$

   (N = # colors, nf = # flavors)

2. *Bare* Polyakov Loops, $\forall$ representations R: factorization for $N \rightarrow \infty$

3. *Renormalized* Polyakov Loops

4. Numerical results from the lattice:

   N=3, nf=0: R = 3, 6, 8 (10?)

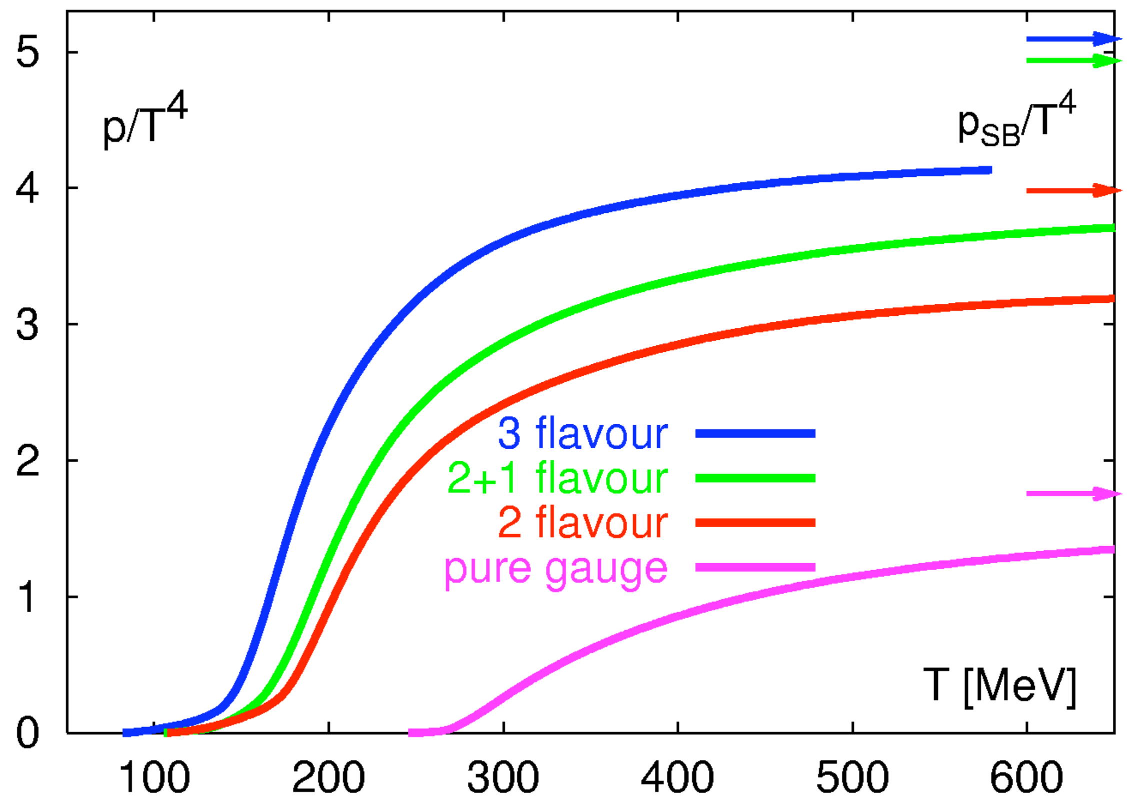
5. Effective (matrix!) model for *renormalized* loops

Review of Lattice Results: N=3, nf = 0, 2, 2+1, 3

nf = 0: T_deconf ≈ 270 MeV

pressure *small* for T < T_d

like N→∞: p ~ 1 for T<T_d, p ~ N^2 for T>T_d (Thorn, 81)



Bielefeld

nf ≠ 0: as nf ↑, p_ideal ↑, T_chiral ↓

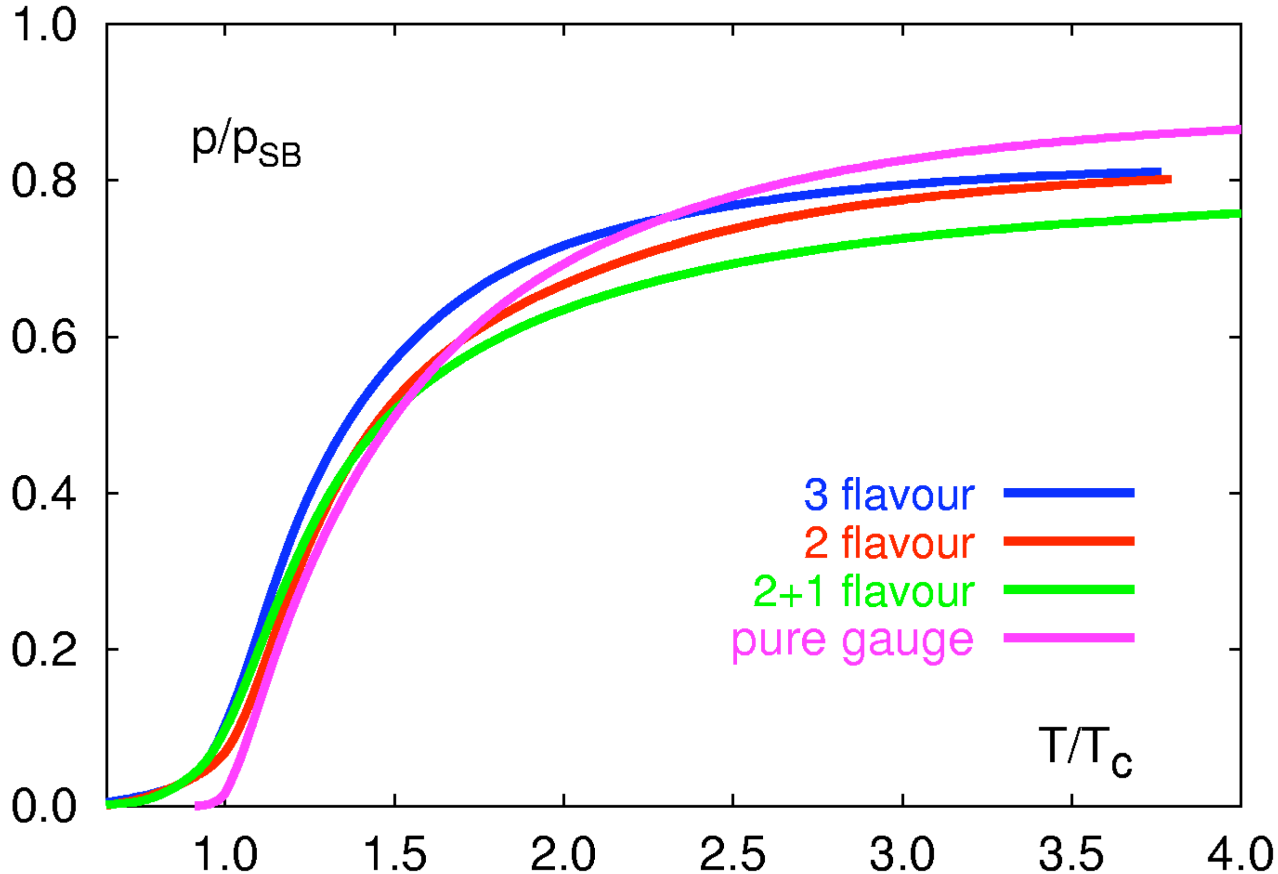
BIG change: between nf = 0 and nf = 3,

p_ideal: 16 to 48.5 x ideal m=0 boson Tc: 270 to 175 MeV!

Even the order changes: first for nf=0 to "crossover" for nf = 2+1

Flavor Independence

$$\frac{p}{p_{ideal}}\left(\frac{T}{T_c}\right) \approx universal$$



Bielefeld

Perhaps: even for nf ≠ 0, "transition" *dominated* by gluons

At T ≠ 0: thermodynamics dominated by Polyakov loops

Three colors, pure gauge: *weakly* first order?

Latent heat: $$\left.\frac{\Delta\epsilon}{\epsilon_{ideal}}\right|_{T=T_d^+} \approx \frac{1}{3}$$ vs 4/3 in bag model. So?

$$\frac{\sigma(T)}{\sigma(0)}$$

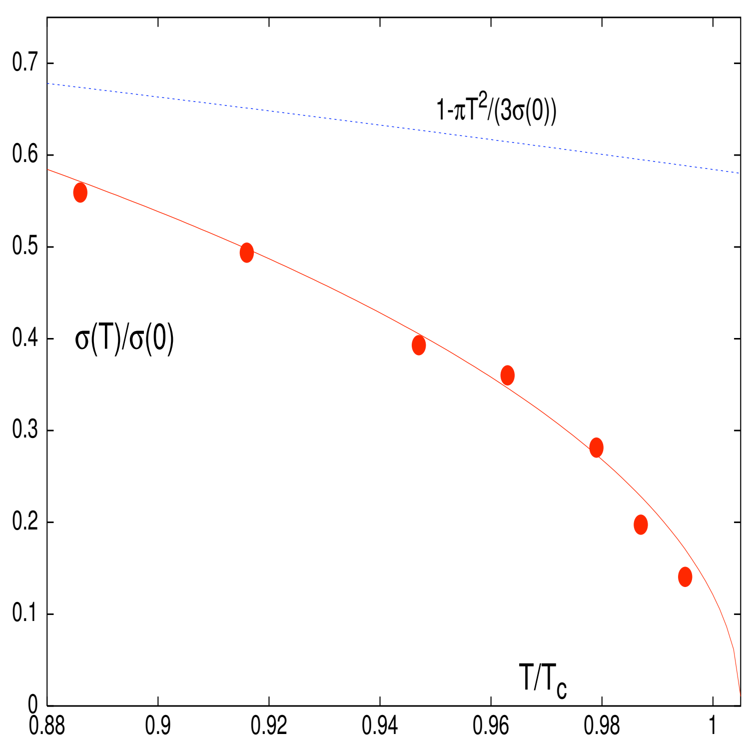


FIG. 1:

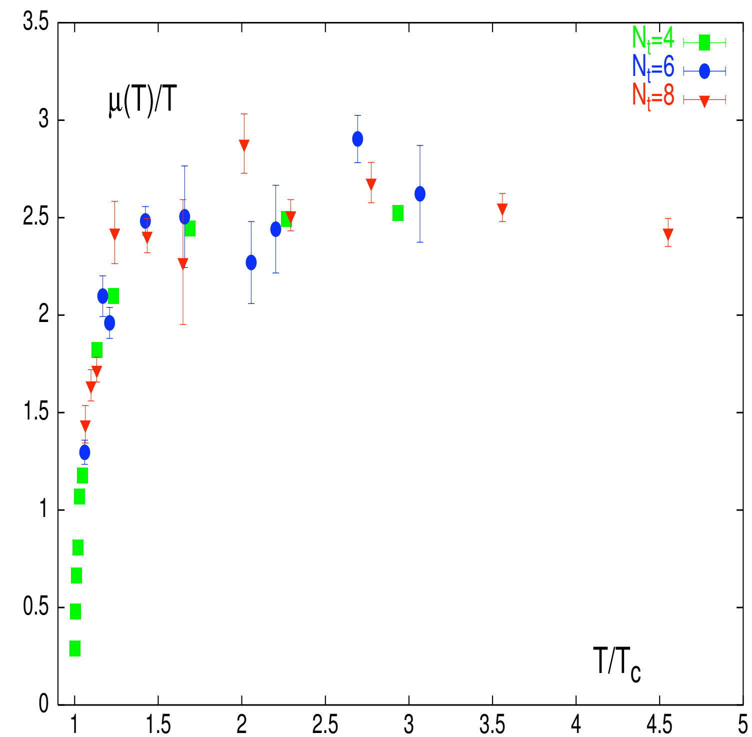


FIG. 1:

$$\frac{m_{Debye}(T)}{m_{Debye}(0)}$$

Bielefeld

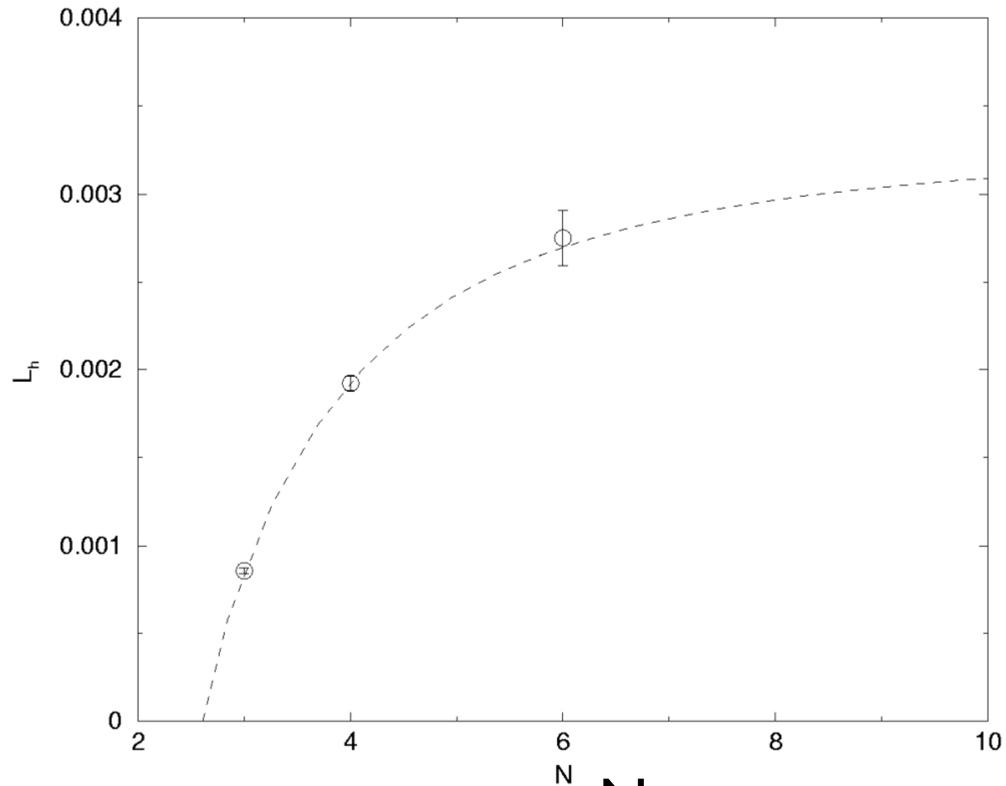
(Some) correlation lengths grow by ~ 10!

$$\frac{\sigma(T_d^-)}{\sigma(0)} \approx \frac{m_{Debye}(T_d^+)}{m_{Debye}(1.5T_d)} \approx \frac{1}{10}$$

Deconfining Transition vs N: First order ∀ N ≥ 4

Lucini, Teper, Wenger '03: latent heat ~ N^2 for N= 3, 4, 6, 8

latent heat/N^2



N

No data for $\sigma(T_d^-)$, $m_{Debye}(T_d^+)$ Is N → ∞ Gross-Witten?

Gross-Witten - First order *but:* $\sigma(T_d^-) = m_{Debye}(T_d^+) = 0$

Wilson Lines at T $\neq$ 0

Always: "pure" SU(N) gauge, *no* dynamical quarks (nf = 0)

Imaginary time formalism: $\tau : 0 \to 1/T$

Wilson line in fundamental representation:

$$\mathbf{L}_N(\vec{x},\tau) = \mathcal{P}\exp\left(ig\int_0^\tau A_0^a(\vec{x},\tau')t_N^a d\tau'\right)$$

= *propagator* for "test quark" at x, moving up in (imaginary) time

$$D_0\mathbf{G}_N = \delta(\tau) => \mathbf{G}_N = \mathbf{L}_N\theta(\tau)$$

$\mathbf{L}_{\overline{N}} = \mathbf{L}_N^\dagger$ = propagator "test anti-quark" at x, moving back in time

$\mathbf{L}_N \epsilon\, SU(N) : \; \mathbf{L}_N^\dagger \mathbf{L}_N = \mathbf{1}_N \;,\; \det(\mathbf{L}_N) = 1$ (Mandelstam's constraint)

Polyakov Loops

Wrap *all* the way around in τ: $\mathbf{L}_N(\vec{x}, 1/T) \rightarrow \mathbf{L}_N$

Polyakov loop = *normalized* loop = gauge invariant

$$\ell_N = \frac{1}{N} \, \mathrm{tr}\, \mathbf{L}_N$$

Confinement: test quarks don't propagate

$$\langle \ell_N \rangle = 0 \quad , \quad T < T_{deconf}$$

Deconfinement: test quarks propagate

$$\langle \ell_N \rangle = e^{i\theta} |\langle \ell_N \rangle| \neq 0 \quad , \quad T > T_d$$

$e^{iN\theta} = 1 :$ Spontaneous breaking of global Z(N) = center SU(N)

't Hooft '79, Svetitsky and Yaffe, '82

Adjoint Representation

Adjoint rep. = "test meson"

$$\mathrm{tr}\,\mathbf{L}_{adj} = |\mathrm{tr}\,\mathbf{L}_N|^2 - 1$$

Note: both coefficients ~ 1

Check: $\mathbf{L}_N = \mathbf{1}_N \to \mathrm{tr}\,\mathbf{L}_{adj} = N^2 - 1$ = dimension of the rep.

Adjoint loop: divide by dim. of rep.

$$\ell_{adj} = \frac{1}{N^2 - 1}\,\mathrm{tr}\,\mathbf{L}_{adj}$$

At large N,

$$\ell_{adj} = |\ell_N|^2 + O\left(\frac{1}{N^2}\right)$$ = *factorization*

Two-index representations

2-index rep.'s = "di-test quarks" = symmetric or anti-sym.

$$\mathrm{tr}\mathbf{L}_{(N^2\pm N)/2} = \frac{1}{2}\left((\mathrm{tr}\mathbf{L}_N)^2 \pm \mathrm{tr}\mathbf{L}_N^2 \right)$$

Di-quarks: two qks wrap once in time, or one qk wraps twice

Again: both coeff's ~ 1. Subscript = dimension of rep.'s = (N^2 ± N)/2

For *arbitrary* rep. R, if d_R = dimension of R,

$$\ell_R \equiv \frac{1}{d_R}\,\mathrm{tr}\,\mathbf{L}_R$$

For 2-index rep.'s ±, as N → ∞,

$$\ell_\pm \sim \ell_N^2 + O\left(\frac{1}{N}\right)$$ corr.'s 1/N, *not* 1/N^2: $\sim \frac{1}{N^2}\mathrm{tr}\mathbf{L}_N^2$

Representations, N=3

Label rep.'s by their dimension:

fundamental = 3

adjoint = 8

symmetric 2-index = 6

special to N=3: anti-symmetric 2-index = $\overline{3}$

"test baryon" = 10:

$$\ell_{10} = \frac{1}{10}\left(\mathrm{tr}\,\mathbf{L}_3\ \mathrm{tr}\,\mathbf{L}_3^2 + 1\right)$$

Measured 3, 6, 8, & 10 on lattice

Loops at Infinite N

"Conjugate" rep.'s of Gross & Taylor '93: $\mathbf{L}_N$ and $\mathbf{L}_{\overline{N}} = \mathbf{L}_N^\dagger$

If all test qks and test anti-qks wrap once and *only* once in time,

$$\mathrm{tr}\,\mathbf{L}_R = \#(\mathrm{tr}\,\mathbf{L}_N)^{p_+}(\mathrm{tr}\,\mathbf{L}_N^\dagger)^{p_-} + \ldots$$

Many other terms:

$$\#'\ \mathrm{tr}\,\mathbf{L}_N^2\ (\mathrm{tr}\,\mathbf{L}_N)^{p_+-2}(\mathrm{tr}\,\mathbf{L}_N^\dagger)^{p_-} + \ldots$$

dimension R = $d_R \sim N^{p_+ + p_-}$

As N → ∞, *if* #, #'... are all of order 1, *first* term dominates, and:

$$\ell_R \sim (\ell_N)^{p_+}\ (\ell_{\overline{N}})^{p_-} + O\left(\frac{1}{N}\right)$$

Normalization: if $\quad \ell_N = \ell_{\overline{N}} = 1\ ,\ \ell_R = 1\ \forall\ R$

Factorization at Infinite N

In the deconfined phase, the fundamental loop condenses:

$$\langle \ell_N \rangle = e^{i\theta} |\langle \ell_N \rangle| \neq 0 \quad , \quad T > T_d \quad , \quad e^{iN\theta} = 1$$

Makeenko & Migdal '80: at N=∞, *expectation values* factorize:

$$\langle \ell_R \rangle = \langle \ell_N \rangle^{p_+} \langle \ell_{\overline{N}} \rangle^{p_-} + O\left(N^{-1}\right)$$
$$= e^{ie_R\theta} |\langle \ell_N \rangle|^{p_+ + p_-} + O(N^{-1})$$

Phase trivial: $e_R = p_+ - p_-$ = Z(N) charge of R, defined modulo N

Magnitude *not* trivial: highest powers of $|\langle \ell_N \rangle|$ win.

At infinite N, *any* loop order parameter for deconfinement, for *all* e_R

For adjoint loop: Damgaard '87 + ...

N.B.: $\langle \ell_{test\ baryon} \rangle = \langle \ell_N \rangle^{N-1} \langle \mathrm{tr}\ \mathbf{L}_N^2 / N \rangle$

“Mass” renormalization for loops

Loop = propagator for infinitely massive test quark.

Still has mass renormalization, proportional to length of loop:

$$\langle \ell_R \rangle = \exp\left(-m_R/T\right) \quad , \quad m_R \equiv f_R^{div}/a$$

a = lattice spacing. m_R = 0 with dimensional regularization, but so what?

To 1-loop order in 3+1 dimensions:

$$\langle \ell_R \rangle - 1 \sim -\left(\frac{1}{T}\right) \mathcal{C}_R g^2 \int^{1/a} \frac{d^3k}{k^2} \sim -\frac{\mathcal{C}_R g^2}{aT}$$

Divergences order by order in g^2. *Only* power law divergence for straight loops in 3+1 dim.'s. ; corrections ~aT.

In 3+1 dim.'s, loops with cusps *do* have logarithmic divergence.

In 2+1 dim.'s, straight loops also have log. div.'s. (cusps do not)

Renormalization of Wilson Lines

Gervais and Neveu '80; Polyakov '80; Dotsenko & Vergeles '80 ....

For *irreducible* representations R, *renormalized* Wilson line:

$$\widetilde{\mathbf{L}}_R = \mathbf{L}_R / Z_R \quad , \quad Z_R \equiv \exp(-m_R/T)$$

Z_R = renormalization constant for Wilson line

As R's irreducible, different rep's don't mix under renormalization

No anomalous dim. for Wilson line: *no* condition to fix Z_R at some scale

For *all* local, composite operators, Z's *independent* of T

Wilson line = *non*-local composite operator:

Z_R temperature *dependent*: from 1/T, and m_R

(numerically, from simulations)

Renormalization of Polyakov Loops

Renormalized loop: $\widetilde{\ell}_R = \ell_R / Z_R$

Constraint for bare loop: $|\ell_R| \leq 1$

For renormalized loop: $|\widetilde{\ell}_R| \leq Z_R^{-1}$

If $m_R > 0 \; \forall \, T, \; Z_R \to 0$ as a → 0, no constraint on ren.'d loop

E.g.: as T → ∞, ren'd loops approach 1 from above: (Gava & Jengo '81)

$$\langle \widetilde{\ell}_R \rangle - 1 \sim - \left(\frac{1}{T} \right) \mathcal{C}_R g^2 \int \frac{d^3 k}{k^2 + m^2_{Debye}} \sim (-) C_R g^2 (-) (m^2_{Debye})^{1/2}$$

$$\langle \widetilde{\ell}_R \rangle \approx \exp \left(+ \frac{C_R}{N} \frac{(g^2 N)^{3/2}}{8 \pi \sqrt{3}} \right)$$

Smooth large N limit: $C_R \approx \dfrac{N}{2} (p_+ + p_-) + O(1)$

Why all representations?

Previously: concentrated on loops in fund. and adj. rep.'s, esp. with cusps.

At T ≠ 0, natural for loops, at a given point in space, to wrap around in

τ many times. Most general gauge-invariant term:

$$\mathcal{G} = (\mathrm{tr}\, \mathbf{L}_{R_1^+}^{q_1^+})^{n_1^+} (\mathrm{tr}\, \mathbf{L}_{R_2^+}^{q_2^+})^{n_2^+} \ldots (\mathrm{tr}\, (\mathbf{L}_{R_1^-}^{\dagger})^{q_1^-})^{n_1^-} \ldots$$

By group theory (the character expansion):

$$\mathcal{G} = \Sigma_R \, c_R \, \ell_R$$

It is *only* consistent to renormalize this expression, linear in the loops.

If all m_R > 0, at a = 0: $\mathcal{G} = c_1$

Discovered numerically. Irrelevant: physics is in the ren.'d loops.

Lattice Regularization of Polyakov Loops

Basic idea: compare two lattices. *Same* temperature, *different* lattice spacing

If a << 1/T, ren'd quantities the same.

N_t = # time steps = 1/(aT) changes between the two lattices: get Z_R

N_s = # spatial steps; keep N_t/N_s fixed to minimize finite volume effects

$$\log\left(|\langle \ell_R \rangle|\right) = -f_R^{div} N_t + f_R^{cont} + f_R^{lat} \frac{1}{N_t} + \ldots$$

$f_R^{div} \to Z_R$ $\quad$ $f_R^{cont} \to \langle \widetilde{\ell}_R \rangle$ $\quad$ Numerically, $f_R^{lat} \approx 0$

Each f_R is computed at fixed T. As such, there is *nothing* to adjust.

N.B.: also finite volume corrections from zero modes; to be computed.

Explicit exponentiation of divergences to ~g^4 at a≠0:

Curci, Menotti, & Paffuti, '85

Lattice Results

Standard Wilson action, three colors, quenched.

$$N_t = 4, 6, 8, \&10. \;\; N_s = 3N_t$$

Lattice coupling constant $\beta = 6/g^2$

β_d = coupling for deconfining transition: $= \beta_d(N_T)!$

Non-perturbative renormalization:

$$\log(T/T_d) = 1.7139(\beta - \beta_d) + \ldots$$

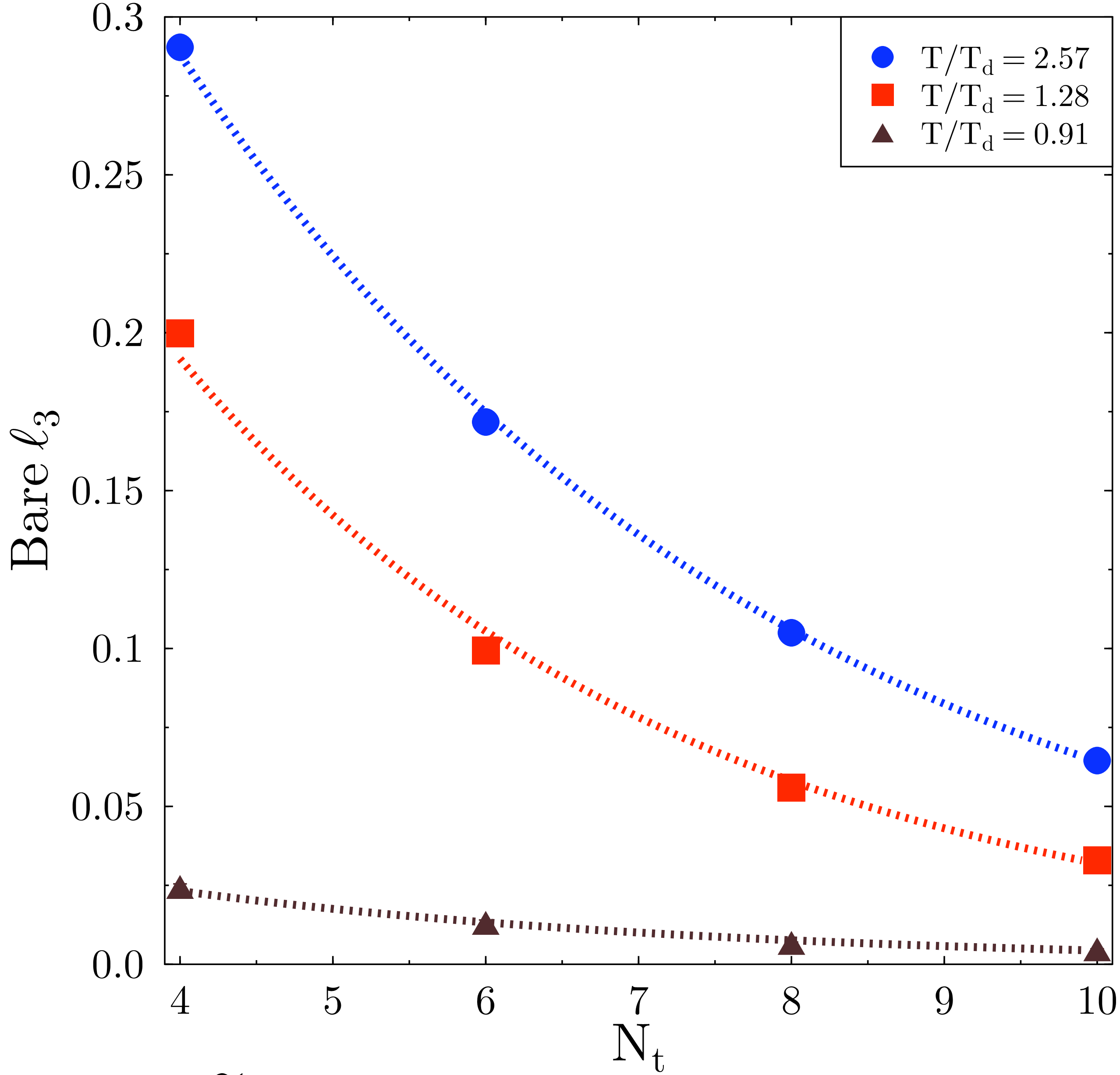
To get the same T/T_d @ different N_t, must compute at *different* β!

Calculate grid in β, interpolate to get the same T/T_d at different N_t

N.B. : Method same with dynamical quarks

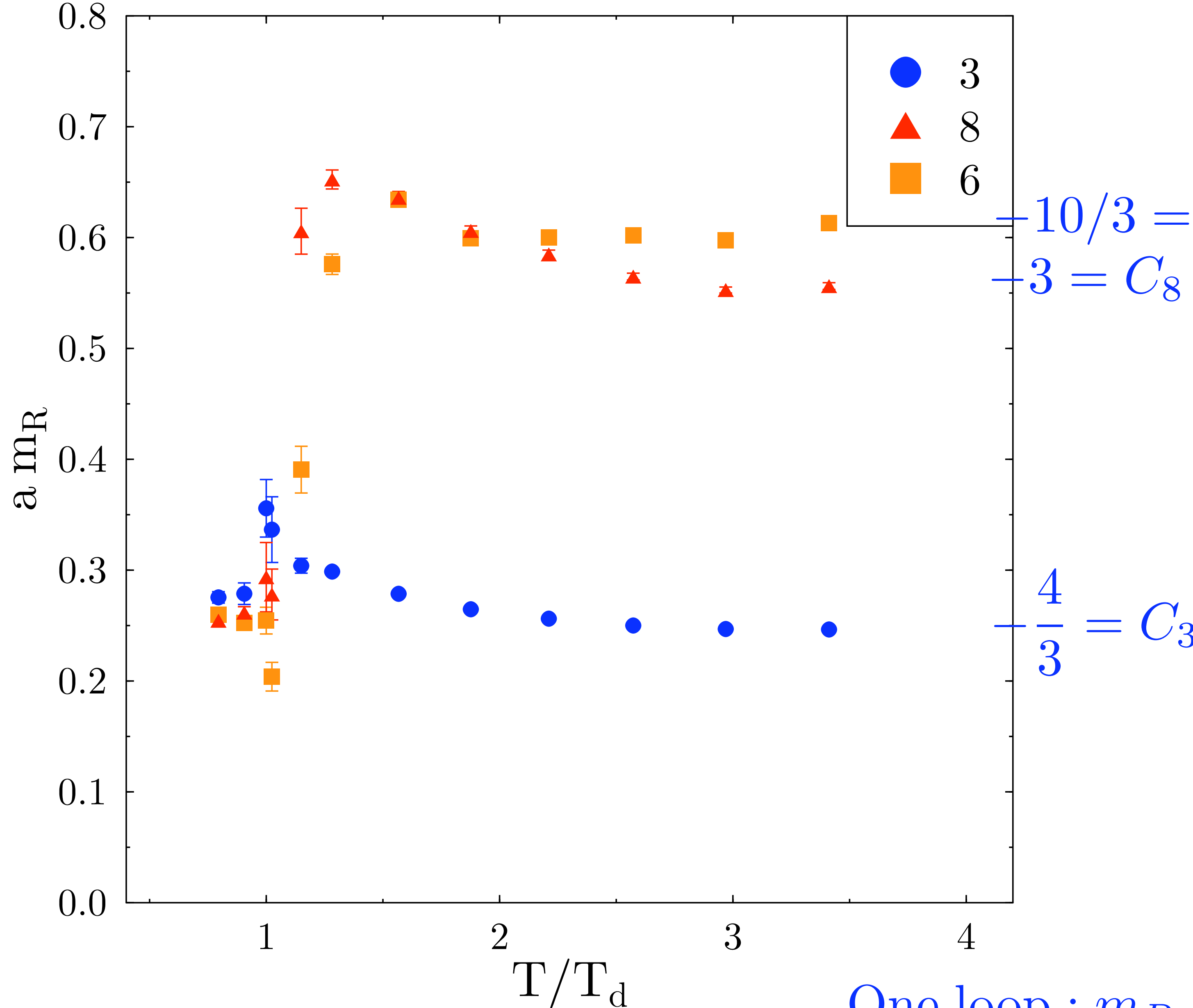
Measured $\ell_3,\ \ell_6,\ \ell_8,\ \&\ \ell_{10}$ (No signal for 10 for N_t > 4)

Bare $|\ell_3|$ vs N_t



$$|\langle \ell_3 \rangle| \equiv \exp(-m_3/T)|\langle \widetilde{\ell_3} \rangle|$$

Divergent mass $m_R(T)$



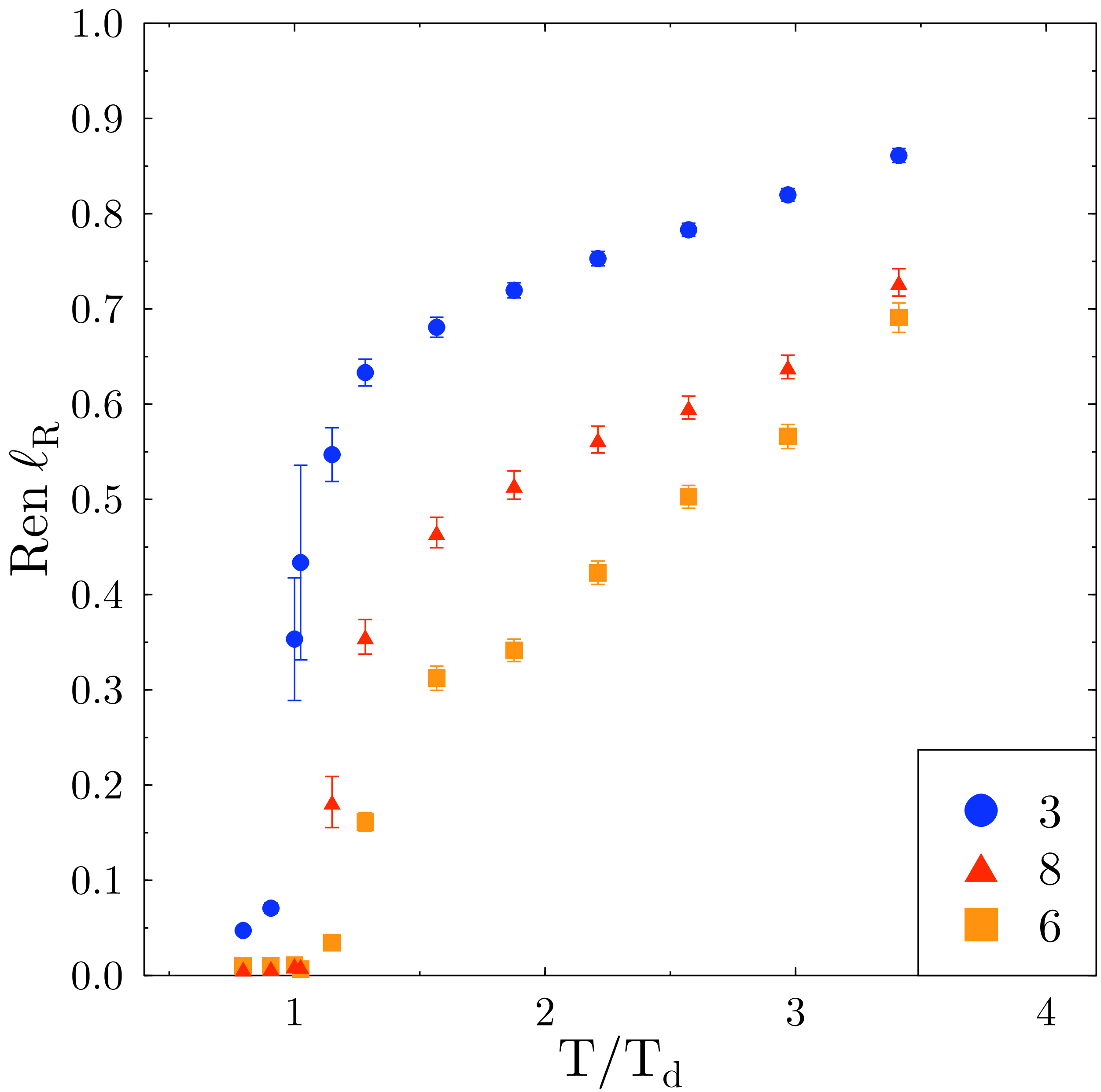
One loop : $m_R \sim C_R$

OK for T > 1.5 T_d;

Fails for $T_d \to 1.5 T_d$: $m_8 > m_6$

N.B. : *all* $m_R > 0 \; \forall \; T$

Renormalized Polyakov Loops



$\widetilde{\ell}_3 > \widetilde{\ell}_8 > \widetilde{\ell}_6$

No signal of decuplet loop at N_t > 4; C_10 big, so bare loop small

Results for Ren'd Polyakov Loops

$T < T_d : Z(3)$ symmetry $\Rightarrow \langle \widetilde{\ell}_3 \rangle = \langle \widetilde{\ell}_6 \rangle = 0$

But $Z(3)$ charge $e_8 = 0 \Rightarrow \langle \widetilde{\ell}_8 \rangle \neq 0$ for $T < T_d$.

$$\text{Numerically} : \langle \widetilde{\ell}_8 \rangle = \; small \; \# \frac{1}{N^2} \approx 0 \; , \; T < T_d$$

Like large N: Greensite & Halpern '81, Damgaard '87...

(Similar to measuring adjoint string tension in confined phase)

Transition first order → ren.'d loops *jump* at T_d:

$$|\langle \widetilde{\ell}_3 \rangle| \approx .4 \pm .05 \; , \; T = T_d^+$$

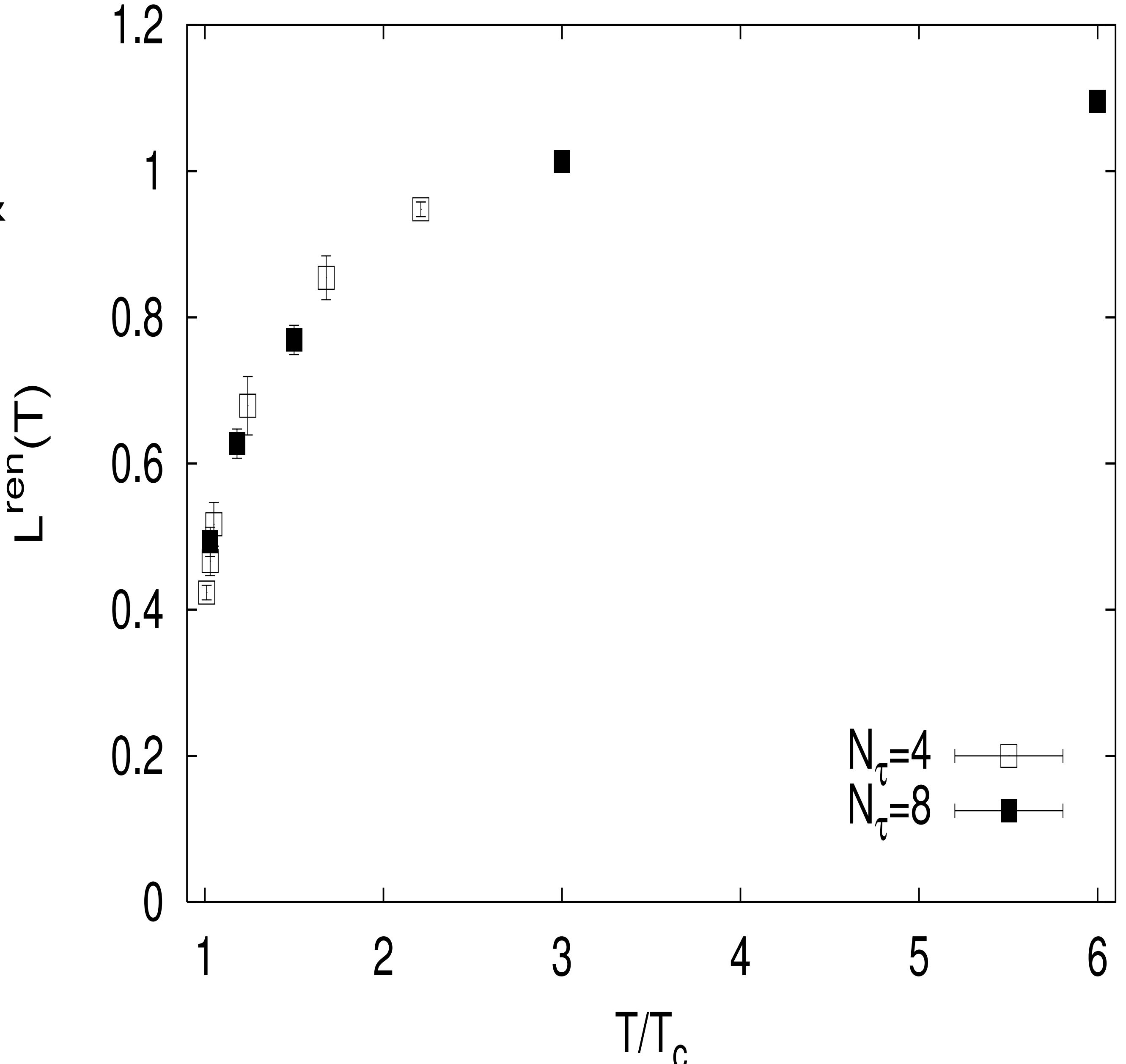
T > T_d: Find ordering: 3 > 8 > 6. But compute difference loops:

$$\delta \widetilde{\ell}_6 \equiv \langle \widetilde{\ell}_6 \rangle - \langle \widetilde{\ell}_3 \rangle^2 \sim O(1/N)$$

$$\delta \widetilde{\ell}_8 \equiv \langle \widetilde{\ell}_8 \rangle - |\langle \widetilde{\ell}_3 \rangle|^2 \sim O(1/N^2)$$

Bielefeld's Renormalized Polyakov Loop

Kaczmarek, Karsch, Petreczky, & Zantow '02



$|\langle\widetilde{\ell_3}\rangle|$

Approx. agreement.

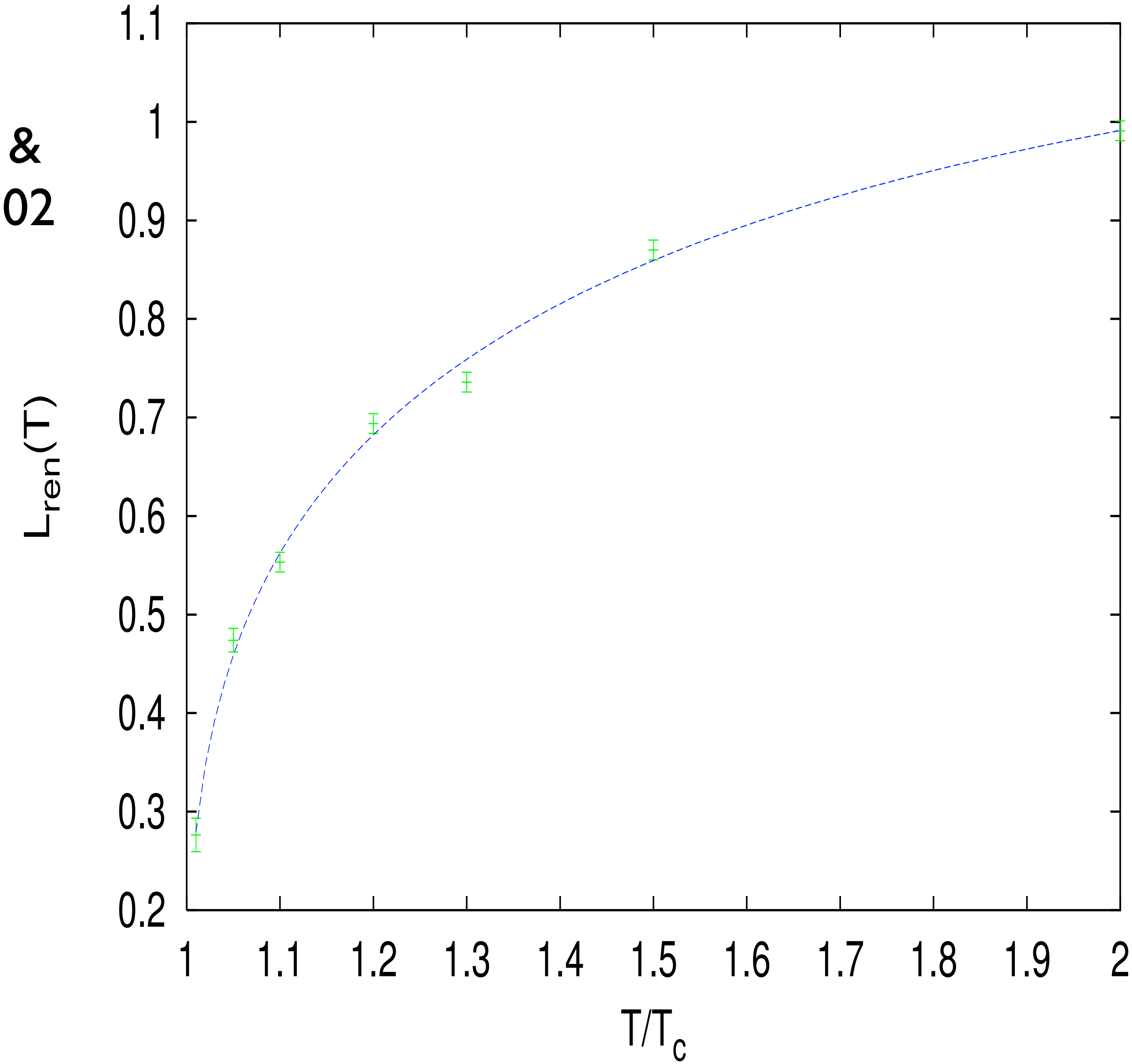
$$\langle \ell_3^*(x)\ell_3(0)\rangle - |\langle \ell_3\rangle|^2 \sim Z_3^2 \exp(-m_{Debye}|x|)$$

Need 2-pt fnc at one N_t vs 1-pt fnc at several N_t

Bielefeld's Ren'd Polyakov Loop, N=2

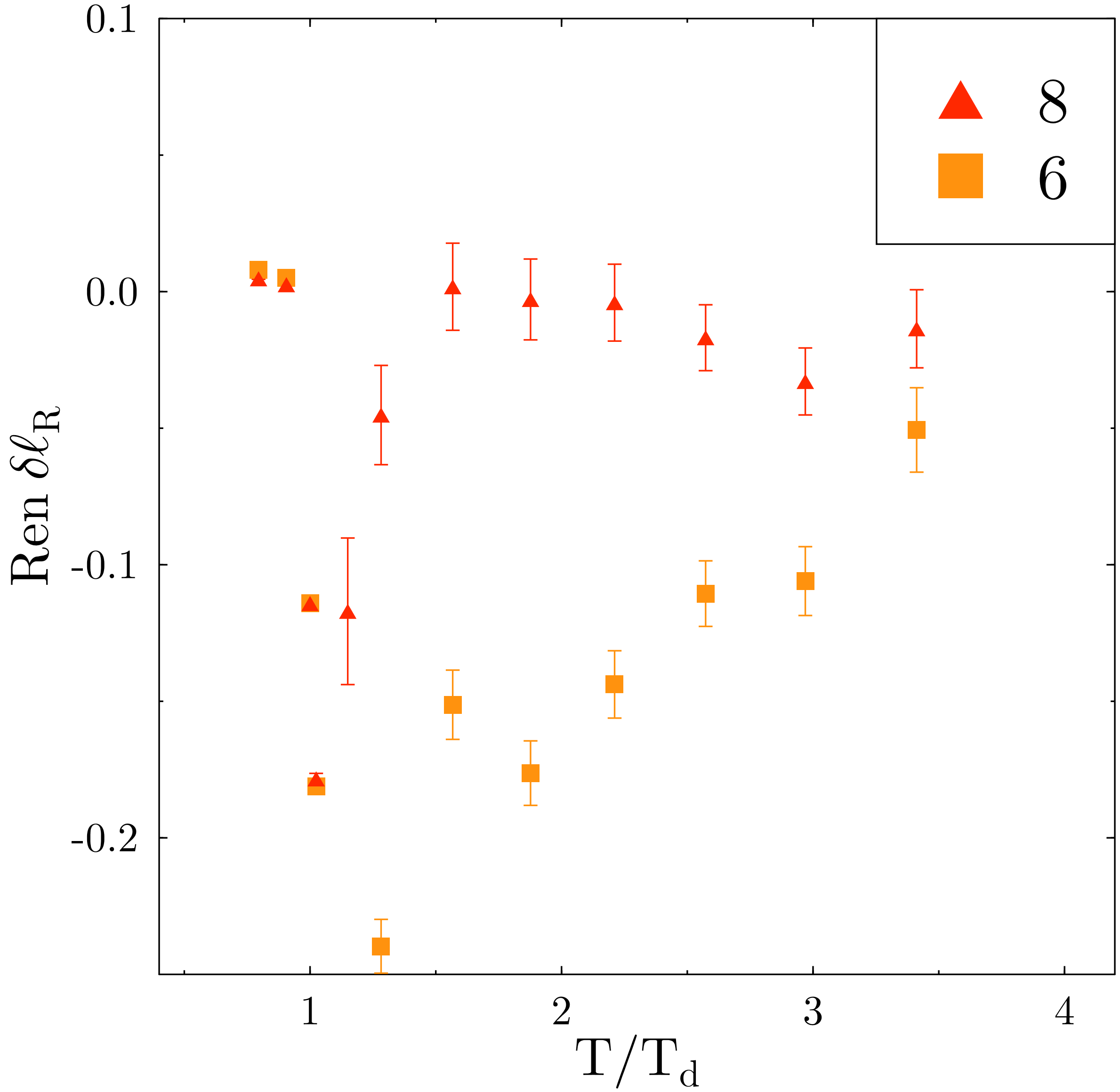
Digal,
Fortunato, &
Petreczky '02

$|\langle \widetilde{\ell_2} \rangle|$



Transition second order: $\Rightarrow |\langle \widetilde{\ell_2} \rangle| = 0 \; @ \; T = T_d^+$

Difference Loops: *Test* of Factorization at N=3



Details of spikes near T_d?

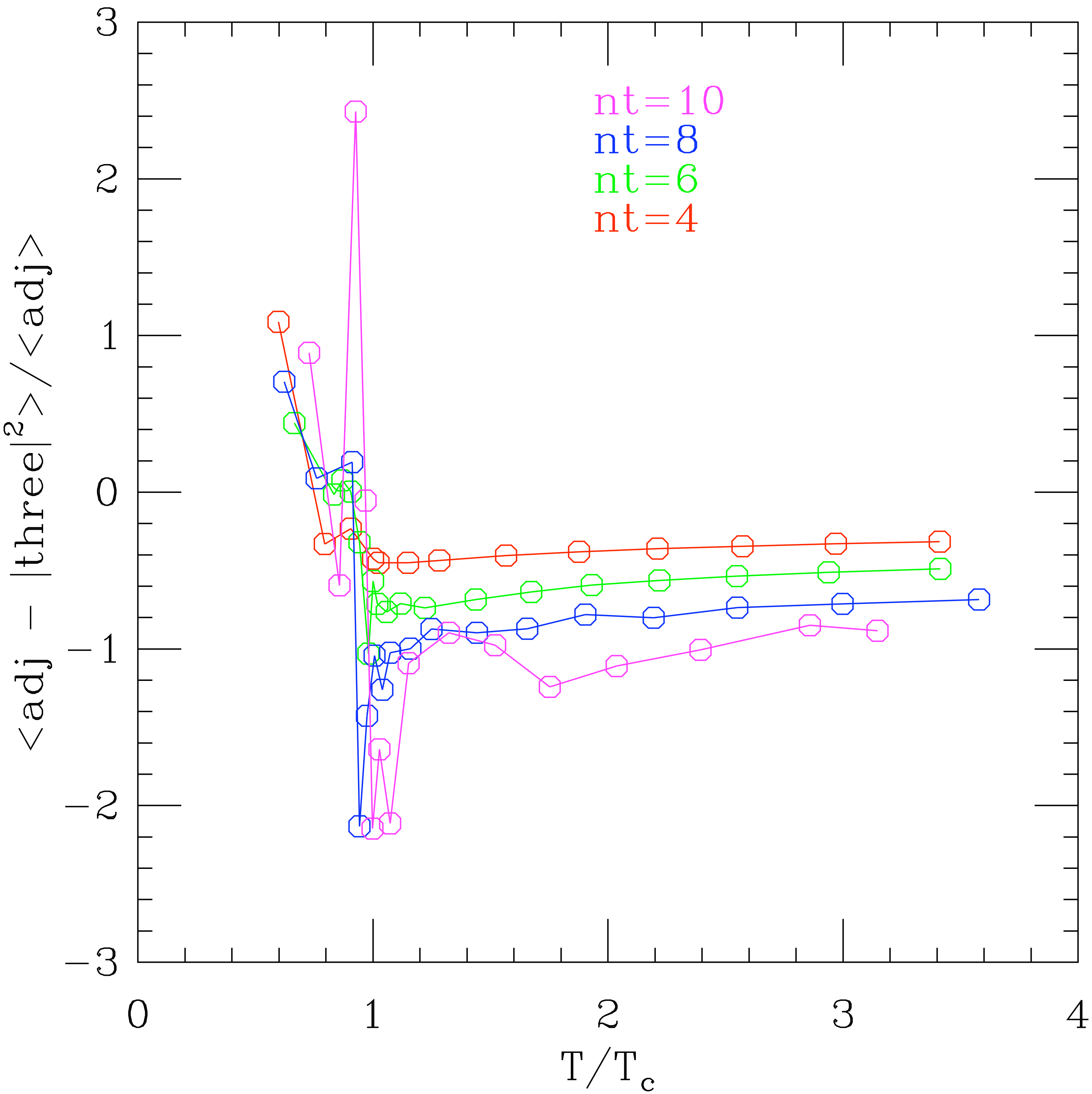
Sharp octet spike

Broad sextet spike

$$|\delta\widetilde{\ell_8}| \sim O(1/N^2) \leq .12 \; ; \; |\delta\widetilde{\ell_6}| \sim O(1/N) \leq .25$$

Bare Loops *don't* exhibit Factorization

Bare octet difference loop/bare octet loop: violations of factor. 50% @ Nt =4 200% @ Nt = 10.



Mean Field Theory for Fundamental Loop

At large N, if fundamental loop condenses, factorization $\Rightarrow$ *all* other loops

This is a mean field type relation; implies mean field for $\langle \widetilde{\ell}_N \rangle$?

General effective lagrangian for *renormalized* loops:

Choose basic variables as Wilson lines, not Polykov loops: (i = lattice sites)

$$\mathcal{Z} = \int \Pi \, d\mathbf{L}_N(i) \; \exp\left(-\mathcal{S}(\ell_R(i))\right) \qquad\qquad \mathbf{L}_N(i) \, \epsilon \, SU(N)$$

Loops automatically have correct Z(N) charge, and satisfy factorization.

Effective action Z(N) symmetric. Potential terms (starts with adjoint loop):

$$\mathcal{W} = \Sigma_i \Sigma_R^{e_R=0} \; \gamma_R \ell_R(i)$$

and next to nearest neighbor couplings:

$$\mathcal{S}_R = -(N^2/3) \Sigma_{i,\hat{n}} \Sigma_{R,R'}^{e_R+e_{R'}=0} \; \beta_{R,R'} \; \mathrm{Re} \; \ell_R(i) \ell_{R'}(i+\hat{n}) \; .$$

In mean field approximation, that's it. (By using character exp.)

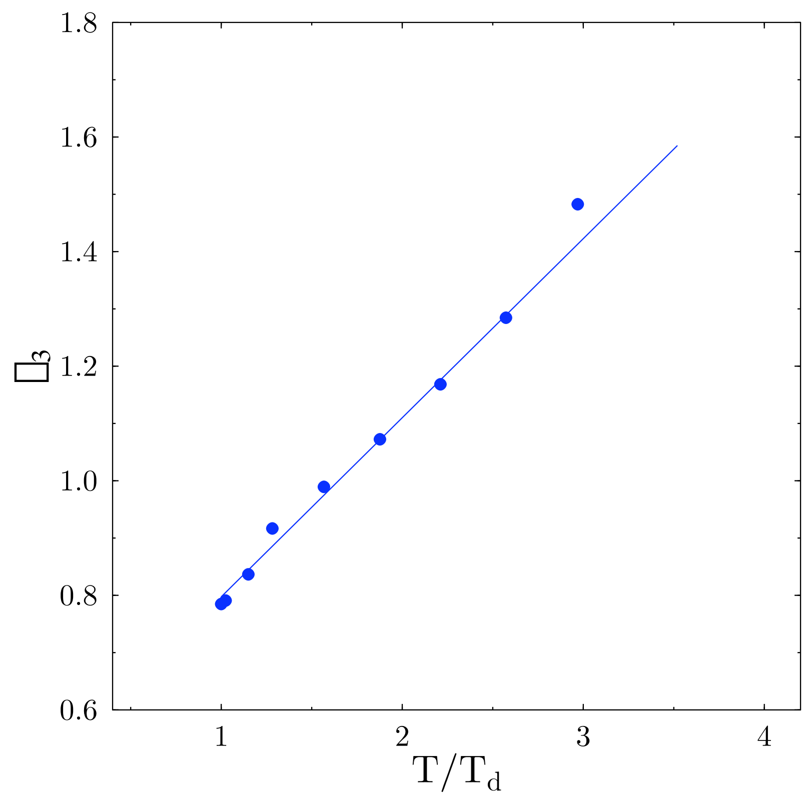
Matrix Model (=Mean Field) for N=3

Simplest possible model: only $\beta_{3,3^*} \equiv \beta_3 \neq 0$ (Damgaard, '87)

$$\langle \ell_3 \rangle = \int d\mathbf{L}\ \ell_3\ \exp\left(18\beta_3 \langle \ell_3 \rangle \mathrm{Re}\ell_3\right) \Big/ \int d\mathbf{L}\ \exp\left(18\beta_3 \langle \ell_3 \rangle \mathrm{Re}\ell_3\right)$$

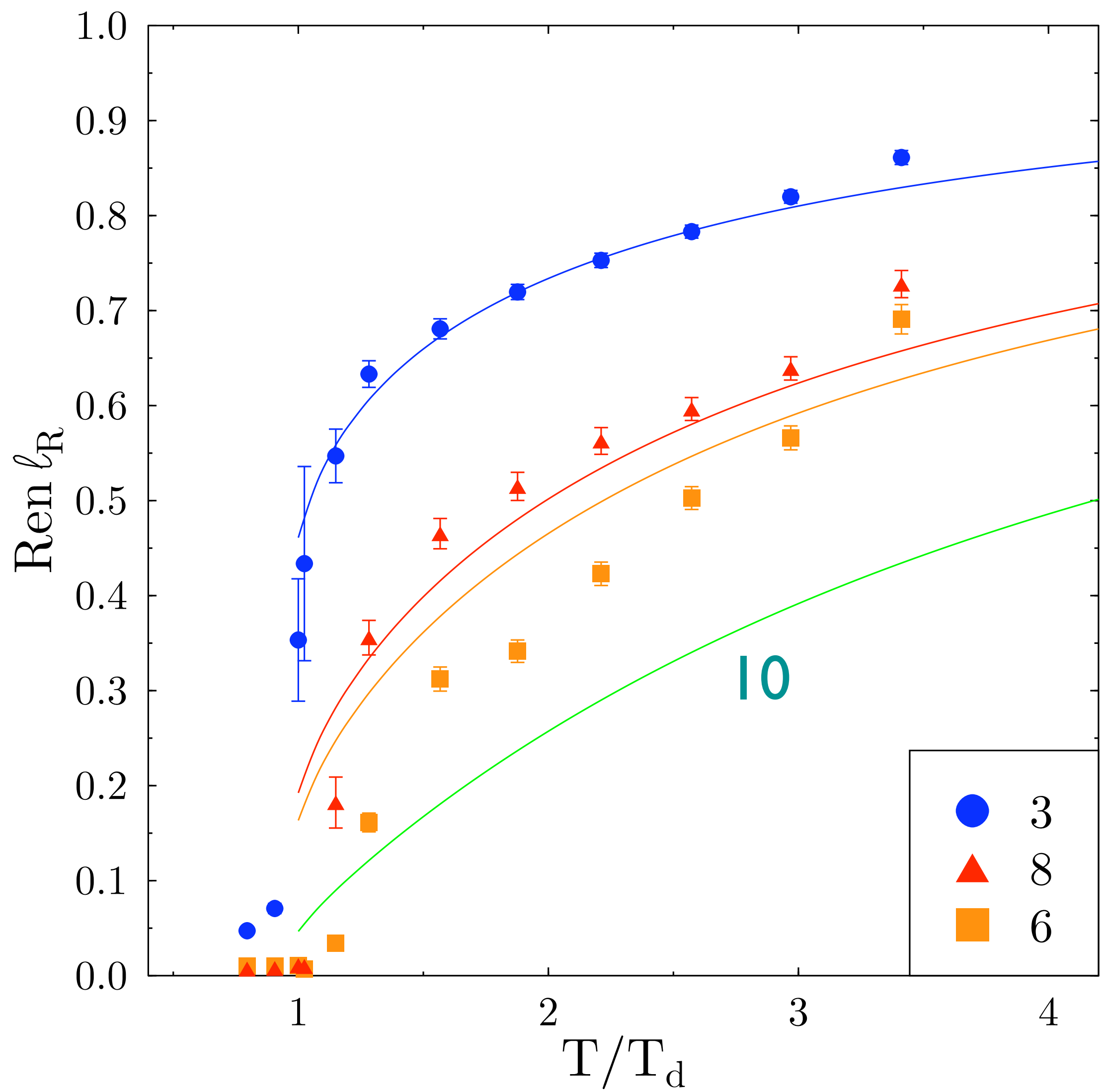
Fit $\beta_3(T)$ to get $\langle \widetilde{\ell_3} \rangle(T)$

Find $\beta_3(T)$ linear in T



Now compute loops in other representations using this $\beta_3(T)$

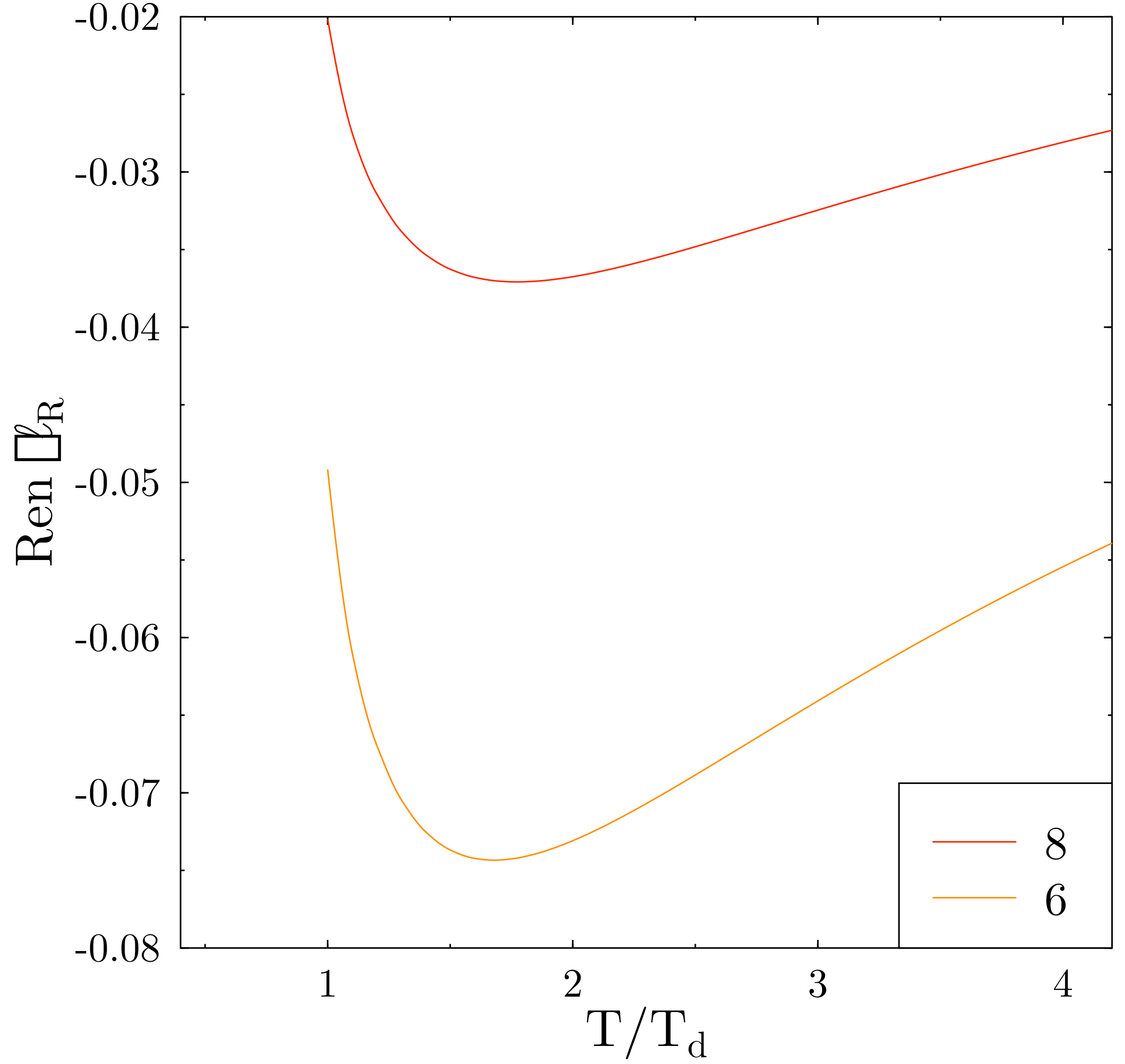
Matrix Model: N=3



Approximate agreement for 6 & 8. Predicts 10 should be there.

Solid lines = matrix model. Points = lattice data for renormalized loops.

Difference Loops for Matrix Model, N=3



Sextet diff. loop > octet, in agreement with 1/N exp.

But much broader, and much smaller, than the lattice data!

Matrix Model, N=∞, and Gross-Witten

Consider mean field, where the *only* coupling is $\beta_{N,N^*} \equiv \beta$

Gross & Witten '80, Kogut, Snow, & Stone '82, Green & Karsch '84

At N=∞, mean field potential is non-analytic, given by two *different* potentials:

$$\mathcal{V}^{-}_{mf} = \beta(1-\beta)\ell^2 \quad , \quad \ell \leq 1/(2\beta)$$

$$\mathcal{V}^{+}_{mf} = -2\beta\ell + \beta\ell^2 + \frac{1}{2}\log(2\beta\ell) + \frac{3}{4} \quad , \quad \ell \geq 1/(2\beta)$$

For fixed β, the potential is everywhere continuous, but its *third* derivative is not, at the point $\ell = 1/(2\beta)$

$\beta \leq 1 : \langle \ell \rangle = 0$ = confined phase

$\beta \geq 1 : \langle \ell \rangle \neq 0$ = deconfined phase

$$\langle \ell \rangle = \frac{1}{2}\left(1 + \sqrt{\frac{\beta - 1}{\beta}}\right) :$$

$$\langle \ell \rangle = \frac{1}{2} \; , \; \beta = 1^+$$

$$\langle \ell \rangle \rightarrow 1 \; , \; \beta \rightarrow \infty$$

Gross-Witten Transition: "Critical" First Order

Transition first order. Order parameter jumps: 0 to 1/2. Also, latent heat $\neq$ 0:

$$\mathcal{V}^{-}_{mf} = 0 \, , \; \beta \leq 1 \quad , \quad \mathcal{V}^{+}_{mf} \approx -(\beta - 1)/4 \, , \; \beta \to 1^{+}$$

But masses vanish, asymmetrically, at the transition!

$$m_{-}^{2} \approx 2(1-\beta) \quad , \quad \beta \to 1^{-} \; . \qquad m_{+}^{2} \approx 4\sqrt{\beta - 1} \quad , \quad \beta \to 1^{+} \; .$$

If β~T, *and* the deconfining transition is Gross-Witten at N=∞, then the string tension and the Debye mass *vanish* at T_d as:

$$\sigma(T) \sim (T_d - T)^{1/2} \, , \; T \to T_d^{-}$$

$$m_{Debye}(T) \sim (T - T_d)^{1/4} \, , \; T \to T_d^{+}$$

Other terms? Adjoint loop in potential just shifts β.

Higher terms in potential do seem to give ordinary 1st order transitions.

Lattice: N=3 close to Gross-Witten. N>3?

To do

Two colors: matching critical region near T_d to mean field region about T_d?

Higher rep.'s, factorization at N=2?

Three colors: better measurements, esp. near T_d: $\langle \widetilde{\ell}_3 \rangle (T_d^+) \ldots$

"Spikes" in sextet and octet loops? Fit to matrix model?

For decuplet loop, use "improved" Wilson line? $\int d\Omega_{\vec{n}} \sim$ HTL's

$$\mathbf{L}_{\mathrm{imp}} = \int d\Omega_{\vec{n}} \; \exp(\, ig \int (A_0 + \kappa a \vec{E} \cdot \vec{n}) d\tau)$$

Four colors: is transition Gross-Witten? Or is N=3 an accident?

With dynamical quarks: method to determine ren.'d loop(s) *identical*

$$\text{Is } \langle \widetilde{\ell}_R \rangle \left(\frac{T}{T_c} \right)_{\textit{with}\ \text{quarks}} \approx \langle \widetilde{\ell}_R \rangle \left(\frac{T}{T_d} \right)_{\text{pure gauge}} ?$$